1 Case Study 1: Portugal[1]

1 Summary of the study

Country	Portugal
Grade (international)	Primary (grade 6)
Subject	Mathematics
Research topic	Introduction of the concept of volume
LS model and variant [link: Chapter 3]	Model: Hybrid LS Variant: School Hybrid LS
Stakeholders [link: 2.3 Stakeholders]	2 pre-service teachers 1 teacher educator 1 school-based mentor
Number of LS sessions and total of hours	9 sessions of 90' (13,5 hours)
Number and type of research lessons	RL1 at school RL2 at school
LS phase: Identify issues in teaching and learning [link: 2.1 Lesson study cycle, phase 0] [link to the description of this phase in the Portuguese case study]	Number of sessions: 1 Choice of the issue: topic chosen from mathematics curriculum for grade 6, according to the school planning. The chosen topic was the exploration of the concept of volume with non-conventional and conventional units.
LS phase: Study material and learning trajectory. Formulate research question [link: 2.1 Lesson study cycle, phase 1]	Number of sessions: 1,5 Material studied: articles Key points of this phase: analysis and anticipation of students' difficulties with the concept of volume, identification of different teaching strategies.

[1] This case study description is based in an article by Nicole Duarte, João Pedro da Ponte and Filipa Faria, submitted for publication. LIFT project thanks Nicole for providing all information about the development of this lesson study.

[link to the description of this phase in the Portuguese case study]	Research question: how can the teacher promote the development of the concept of volume and the measurement of volume?
LS phase: Plan research lesson [link: 2.1 Lesson study cycle, phase 2] [link to the description of this phase in the Portuguese case study]	Number of sessions: 1,5 Key points of this phase: the pre-service teachers selected the tasks for the students, solved the tasks and anticipated students solving strategies. Then they created a lesson plan with the tasks, students' solving strategies, main difficulties, possible teacher interventions and evaluation. Structure of the research lesson(s): exploratory approach. Presentation of the task, autonomous work of the students, whole class discussion and synthesis.
LS phase: Teach and observe lesson [link: 2.1 Lesson study cycle, phase 3] [link to the description of this phase in the Portuguese case study]	Number of sessions: 2 Research lesson(s) taught by: the pre-service teachers Key points of this phase: the research lesson was taught in the school, by one of the pre-service teachers. All the other stakeholders observed it. A table with a list of elements to observe was given to all the observers. Videorecording was made.
LS phase: Reflect on lesson [link: 2.1 Lesson study cycle, phase 4] [link to the description of this phase in the Portuguese case study]	Number of sessions: 3 Stakeholders attending: pre-service teachers, teacher educator Role of the stakeholders: Key points of this phase: This phase consisted of one post-lesson discussion after each research lesson, and a final reflection section. For the post-lesson discussion, there was a first assessment made by the teacher of the lesson, followed by a reflection from the other pre-service teacher and questions from the teacher educator. The pre-service teachers analysed significant episodes and unforeseen events, using their field notes and parts of the videorecording. The pre-service teachers were also asked to produce a written reflection about the lesson and their own learning. After the first research lesson, the second research lesson was refined.
LS phase: Share LS report [link: 2.1 Lesson study cycle, phase 5]	Number of sessions: 2 sessions scheduled (done or not?) Key points of this phase: pre-service teachers described each phase of the lesson study from their perspective, highlighting the main ideas that they learned and the challenges they faced.

[link to the description of this phase in the Portuguese case study]	

1.2 Institutional context and constitution of the group

This lesson study was carried out in Portugal in 2025 at a Higher School of Education in which there is an initial teacher education program to prepare primary teachers (to teach grades 1-6). This program includes a bachelor's degree (3 years) followed by a master's degree (2 years). In this program, the pre-service teachers in this lesson study are prepared to teach all subjects in grades 1-4 and mathematics and science in grades 5-6.

The lesson study involved two pre-service teachers (we will name them Júlia and Alda) who were attending the second semester of the second year of their master's degree program. They were doing a practicum supervised by a teacher of the Higher School of Education (that we will name Marta) and constituted a group set up by the institution. The pre-service teachers were assigned to a grade 6 class with a mentor teacher and students between 12 and 13 years old. Assuming that lesson study would be a rich formative experience for the two pre-service teachers, Marta invited them to carry out this activity. Before beginning the lesson study, Marta explained to the mentor teacher what lesson study consisted of and requested her permission to proceed with the study in her class. The mentor teacher from the school where the practicum took place did not create difficulties for the lesson study but did not want to get much involved due to her limited agenda and also because they have several other requests of a different nature which would coincide with the sessions. Marta and the pre-service teachers drafted a request for authorization from the students' legal guardians, explaining the purpose of the study and providing information about the data collection instruments. All legal guardians authorized the students' participation and the recording of their image during the research lesson. When handing in the request for authorization, the pre-service teachers and the mentor teacher explained to the students that they would be participating in a study, which meant that their lessons would be recorded and observed, but, regardless of this, they should abstract themselves from the camera and maintain their usual behavior.

Marta prepared in advance and facilitated each lesson study session. This preparation included organizing the agenda, assigning tasks for each session, and selecting documents for the pre-service teachers to read. In addition, the pre-service teachers were constantly encouraged to intervene, sharing their ideas and experiences and making suggestions about the preparation of the research lessons. In the post-lesson discussion, Marta essentially took on the role of questioner and mediator of the pre-service teachers' reflections.

Both pre-service teachers had already participated in two lesson studies carried out during the first and second semesters of the previous academic year, with Júlia leading the research lessons. So, this lesson study built on the previous experience of all participants in this kind of work in which quite friendly and collaborative relationships had developed. The overall plan for the lesson study sessions is presented in Table 1. From the very beginning the group decided that each pre-service teacher would teach a research lesson. The sessions took place weekly and lasted for about 90 minutes.

TABLE 1 - STAGES OF THE LESSON STUDY AND THE CORRESPONDING SESSIONS.

Stages of the lesson study	Sessions (S*n*)
Goal setting	S1
Lesson planning	S2, S3, S4
Research lesson 1	S5
Post-lesson discussion 1	S6
Research lesson 2	S7
Post-lesson discussion 2	S8
Final reflection	S9

The two research lessons were led sequentially, with a two-day interval as stipulated by the students' class schedule. For this reason, the mathematical tasks proposed to the students in the two lessons consisted in a meaningful didactic sequence. After each research lesson, a post-lesson discussion session took place. In this session, the pre-service teachers shared the data collected during the lesson, reflected on it, and made suggestions for improvement for the next lesson.

The lesson study was carried out within the scope of the pedagogical practicum, which already includes hours of lesson supervision and group monitoring and reflection between pre-service teachers and the university supervisor. The lesson study sessions were therefore included in these moments, so that pre-service teachers would not be overload with the tasks of lesson study in addition to their usual practicum. The pre-service teachers also used the data collected in the lesson study to prepare their final master's reports.

1.3 Defining the teaching and learning issue **[Link to General definition]**

As this lesson study is embedded in a practicum, Marta decided that the teaching and learning issue should be to teach a well-defined topic from the curriculum. She assumed that teaching curriculum topics is one of the primary tasks of teachers and the pre-service teachers should reinforce their competences in this regard.

The group begun by analyzing the timetable and school planning, and decided that the research lessons would focus on the theme of geometry and measurement and the introduction of the topic of volume. The learning aim of the lesson 1 was to explore the concept of volume and the measurement of the volume of an object using non-conventional units. The learning aim of the lesson 2 was to further explore the concept of volume and the measurement of the volume of an object using non-conventional and conventional units. Thus, the decision was to address a curriculum topic taking into account the curriculum and calendar. It was decided that two lessons would be prepared to be taught in sequence, the first by Júlia and the second by Alda. The pre-service teachers were already familiar with the students of this class since they had observed lessons taught by the mentor teacher and had already been teaching them during the previous semester.

1.4 Study phase and formulation of research question [Link to General definition]

Once the learning goals for the research lessons had been defined, the pre-service teachers consulted articles on teaching and learning the mathematical topic, especially regarding the main difficulties that students usually have in learning it. These documents were discussed in group, which allowed the pre-service teachers to anticipate students' possible difficulties in a more consistent way:

> Júlia: The first thing they mention in the text is that students tend to have difficulty measuring because they cannot think of the unit as part of a whole.
> Alda: What I found important is that most of the texts mention that students of these ages have many difficulties in the issue of spatial visualization of solids.
> Júlia: Yes, and that's why they can't indicate the number of cubes that make up a construction.
> Teacher educator: What about the use of formulas?
> Júlia: Yes, it says here [in the text] that "students who do not understand the processes which lead to formulas for calculating areas or volumes, apply them incorrectly or are not able to solve new problems". And speaking of area, I think they can really confuse area with volume. (S1)

Once the pre-service teachers knew the main difficulties students had in learning the topic, they studied the strategies they could use to make their students' learning more meaningful:

> Júlia: The authors say that the teacher has to understand that it is natural for students to make mistakes and get confused, but that it is part of the learning path for them to understand the concepts involved; they must understand and know the most common strategies used by students in the construction of the visual perception of 3D cubes; and to give students opportunities to repeat and have varied experiences, for the construction of this concept. (S1)

Júlia systematized these three strategies, based on her reading of articles on the teaching of volume, which the teacher can use to support students in the development of their learning of the concept and to help them overcome the difficulties previously identified.

In several courses of the teacher education program and in the previous lesson studies that the pre-service teachers participated, they had opportunity to learn about the national curriculum aims and content and the exploratory curriculum approach, which is similar to the inquiry-based approach and to structured problem-solving lessons. In an exploratory lesson, the teacher challenges students to solve tasks using their previous knowledge and provides opportunities for them to take an active role in the learning process. This lesson, based on a strong teacher-pupil relationship, promotes the interaction between pupils, based on the mathematical content they are exploring. For an exploratory lesson to be productive and sustained by the work and interventions of the pupils, the teacher's activity is essential. For this reason, the pre-service teachers also had opportunity to learn about the role of tasks as a basis for pupil learning, the teaching materials that can be used, the structure of lessons in several phases, the strategies and difficulties of pupils, the way to lead classroom communication, in particular how to organize and lead a whole-class discussion. The research question for this lesson study follows from the definition of the aims of the lessons: how can the teacher promote the development of the concept of volume and the measurement of volume?

1.5 Plan research lesson **[Link to General definition]**

All the work carried out for the research lessons was based on an exploratory approach[2]. In this approach, the teacher begins by presenting a task that is meant to be the starting point for students' work. In introducing the task, the teacher may lead a brief discussion about the meaning of words that the students

[2] A more detailed presentation of the exploratory approach may be found in Ponte and Quaresma (2016).

may not know and recall some concepts that the students have previous learned and that may be important for solving the task.

Then, there is a period of students' autonomous work, in which they work individually, in pairs or small groups. The teacher monitors the students' work and provides assistance to students or groups that have questions or are stuck in their work, doing so by asking questions and taking care of not providing hints with specific strategies to solve the task.

When all students had opportunity to do substantial work on the task, the teacher leads a whole-class discussion, asking the students to present their solutions to the class. The solutions to be presented must be carefully selected and sequenced so that the discussion is productive and leads to meaningful mathematical learning. The students are encouraged to comment the solutions of their colleagues, justifying their statements and arguing with each other.

The lesson ends with a synthesis of the main learning points[3] .

The planning of the lessons 1 and 2 begun by the selection of tasks. The pre-service teachers selected the tasks shown in Figures 1 and 2.

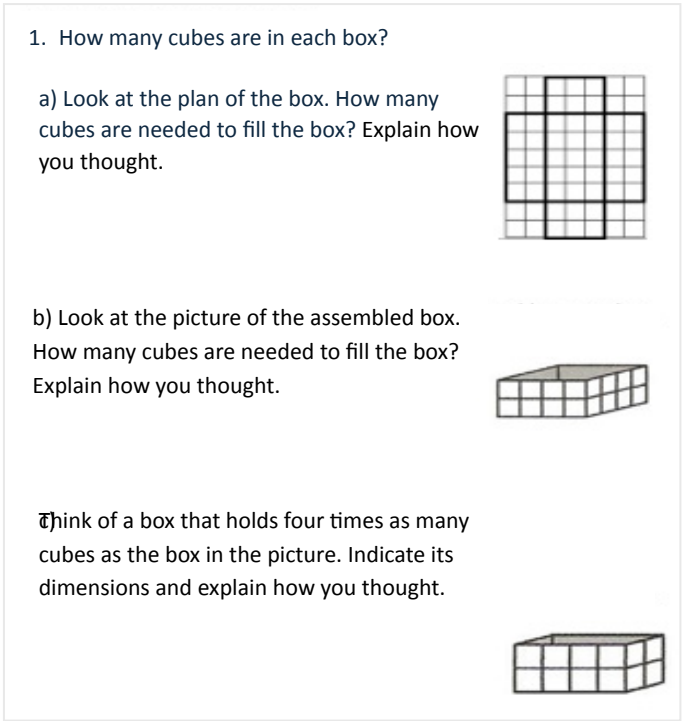
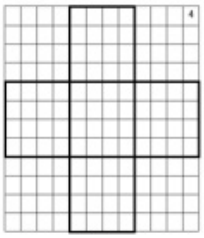


FIGURE 1. TASK PROPOSED TO THE STUDENTS IN RESEARCH LESSON 1.

[3] A detailed presentation of the work of the teacher in preparing and leading a whole class-discussion is provided by Stein et al. (2008).

2. Using one unit of measurement at a time, how many units are needed to fill the box? Write your response in the table.



	Number of units needed to fill the box
	
	
	

3. Imagine a cube with an edge of 8 cm.
a) What is the maximum number of cubes with an edge of 1 cm that it can contain? Explain how you thought.
b) What is the maximum number of cubes with an edge of 2 cm that it can contain? Explain how you thought.

FIGURE 2. TASK PROPOSED TO THE STUDENTS IN RESEARCH LESSON 2.

The pre-service teachers solved the tasks, anticipating different strategies that the students could use:

> Júlia: You can count one by one to arrive at the total number of cubes needed.
> Alda: I counted by layers, that is, like this [points to a layer] and add, or multiply.
> Júlia: Or faces, there are three strategies. They can also go by the formula, if they remember... (S3)

The pre-service teachers included these different strategies for solving the tasks in their lesson plans, as well as the students' possible difficulties (Table 2).

TABLE 2 – RESEARCH LESSON PLAN (TASK 1)

Task	Solving strategies	Students' difficulties	Selection and sequencing of strategies	Teacher interventions
	Unit cube count Counting cubes by rows or columns Counting the cubes per layer (horizontal or vertical), counting 1.A) one by one or calculating the area and adding the cubes from the various layers Calculating the area and multiplying by the height	-Calculate area, not volume -Spatial visualization -Calculation errors -Using different units of measurement -Not understanding the part-whole relationship	Wrong, incomplete, complete Increasing degree of complexity: follow the order in which the solving	- 'You can use drawings or any other form you prefer to explain how you thought.' - 'How did you think?' - 'Can you explain it in your own words?' - 'Don't we know any other way of saying this?'

	Calculating the product of the measures of 3 edges	-Incorrect application of the formula because they do not understand it -Lack of knowledge about the internal structure of the geometric solid -Counting faces, forgetting the inner cubes/not counting the cubes that are not visible -Not counting the cubes that are not visible -Indicating the number of cubes and not the dimensions of the box	strategies are presented (2nd column)	- 'How did you think? Can you explain it in your own words? You can use drawings or any other way you prefer to explain how you thought.' - 'Group A, do you agree with group B?' (knowing a priori that they do not) - 'What do these answers have in common?', 'The result is always the same, but no-one did it the same way... Group A, explain the difference between this answer and that one'. - 'How did you count the cubes?' - 'Are there other ways of counting, group x?' - 'That group counted in another way. Explain how you thought.'
1.B)	Unit cube count Counting cubes by ~~rows or columns Counting the cubes~~ per layer (horizontal or vertical), counting one by one or calculating the area and adding the cubes from the various layers Calculating the area and multiplying by the height			
	Calculating the product of the measures of 3 edges Pictorial ~~representation: drawing the box~~ (assembled or planning)/~~one side of the cube/boxes next to each other~~			
1.C)	Counting cubes (per row/column/layer) Multiply the number of cubes in the box by 4			
	Multiply each dimension of the box by 4			

During the students' autonomous work Júlia and Alda chose to select for the whole-class discussion wrong, incomplete and complete answers. They also agreed to sequence the answers according to their order of correction and to consider their increasing degree of complexity, so that the whole-class discussion was organized as shown in the second column of Table 2.

Finally, the pre-service teachers also planned some interventions (Table 2) to ensure that they asked the students relevant questions, to guide them without influencing them and to encourage students to clarify their reasoning or develop their answers.

To promote the final synthesis, the pre-service teachers decided, in research lesson 1, to elaborate the definition of the concept of volume and, in research lesson 2, to fill in the table in Figure 3. In both cases, the collaboration of the students was sought.

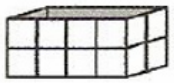
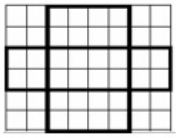
	Units	
	Conventional	Non-conventional
		
		

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FIGURE 3 – SUMMARY TABLE USED IN THE FINAL SYNTHESIS OF THE RESEARCH LESSON 2.

Since Júlia and Alda were already familiar with the exploratory approach, they continued planning the research lessons, now considering the time allocated to each phase of the lesson, as well as its description (Table 3).

TABLE 3 - RESEARCH LESSON PLAN 1.

Theme	**Topic**	**Subtopic**
• Mathematical skills • Geometry and Measurement	• Problem solving • Mathematical communication • Figures in space	• Strategies • Expressing ideas • Volume

Learning goals	**Task/Methodology**	**Resources**	**Evaluation**
• Respect yourself and others; • Being perseverant in face of difficulties; • Develop processes leading to the construction of products and knowledge (C); • Consolidate and deepen the skills you already have, from a lifelong learning perspective (F); • Apply and adapt different problem-solving strategies; • Describe your way of thinking about mathematical ideas and processes, orally and in writing; • Solve tasks that promote exploring the concept of volume and measuring the volume of an object.	***Introduction of the lesson (5')*** The lesson begins with the projection of the introduction of the lesson and its registration by the students in their notebooks. ***Organizing the groups and the classroom (5')*** The teacher explains to the students that they are going to solve a task and informs them of the group formation, asking them to take their seats and helping them to organize the tables and chairs. ***Introduction of the task (5')*** The teacher distributes a statement and the corresponding point 1a) of Task 1 to each group. The teacher reads the statement and point 1a) of Task 1 aloud and the students follow along in their seats. Together with the teacher, the students interpret what the task is about and the teacher clarifies any questions the students may have, making sure that everyone has understood the statement. The teacher informs the students that they will be solving the task for 10 minutes, projecting the timer on the board. ***Students autonomous work (10'×3)*** The groups interpret the task, discuss ideas and strategies for solving it and record them on the worksheet. Meanwhile, the teacher circulates among the different groups, observing and monitoring their work. She gives guidance without leading the students to answers, does	- 17 tasks; - Students' daily notebook; - Computer; - Projector; - Board; Students' writing material; - Board markers.	Learning regulation: formative assessment and interactive regulation.

	not correct errors or incomplete solutions and questions the students to understand their reasoning. The teacher selects the students' strategies that are most relevant to the discussion, taking into account the aim of the lesson, and sequences them. ***Whole-class discussion (10'×3)*** The teacher divides the board according to the number of strategies selected, numbering them. She calls on the students who are going to present the solution strategy, placing them on the board in the anticipated order. These students copy their responses onto the board, explaining their reasoning orally to the class. In the event of a disagreement, the teacher allows the students to compare strategies and discuss faulty or incomplete solutions. The teacher promotes connections between the strategies and between them and the lesson aim. ***Final synthesis (10')*** Under the teacher's guidance, the students construct the definition of volume. This definition is recorded on the board by the teacher and in the daily notebook by the students. ***Recording the summary (5')*** The teacher asks the students what they have done in class and, together with them, writes the summary on the board, while the students record it in their notebooks.		

After considering the lesson registration and the organization of the groups and the classroom, the pre-service teachers organized the lesson into four moments: 1) the introduction of the task, with a planned duration of 5 minutes; 2) the students' autonomous work, with 10 minutes for each part of the task; 3) whole-class discussion, with 10 minutes per part of the task; and 4) the final synthesis, with a planned duration of 10 minutes. The lesson ends with the summary.

1.6 Teach and observe the lesson **[Link to General definition]**

Júlia taught the first research lesson. The observers were Alda, Marta and the mentor teacher. Observing the research lessons allowed the observers to share relevant empirical material collected during the lessons. During the research lesson and with the aim of collecting relevant empirical material, the observers made written records in field notes and also completed the observation table shown in Table 4.

TABLE 4 - OBSERVATION TABLE.

Date:	**Observer:**	**Grade 6**
Aspects to observe	**Observations**	**Comments/notes**
Presentation of the task		

Student reactions		
Student comments		
Students' questions		
Questions to the teacher		
Strategies used by students		
Students' mistakes		
Students' difficulties		
Teacher interventions		
Presentation of students' answers		
Selection of students' answers		
Sequencing students' answers		
Connections between students' answers		
Connections between students' answers and the lesson aim		
Recording on the board		
Situations of disagreement		
Final synthesis		
Exploratory lesson		
Students' learning		
Relevant aspects/episodes to reflect on as a group		
Other		

This table was filled in by each observer during the research lesson and supplemented with field notes which were then shared with the group in the post-lesson discussion and final reflection sessions.

1.7 Reflect on lesson **[Link to General definition]**

Structure of the post-lesson discussion

Marta planned that the post-lesson discussion would be carried out in the following way:

1. The teacher makes a general assessment of the lesson's flow, comparing it with what was planned, identifies and reflects on unforeseen incidents, and makes an assessment of the learning achieved (or not) by the students in relation to the formulated aims, supporting the claims with empirical evidence.
2. The observer (the other pre-service teacher) reflects on the unforeseen incidents of the lesson, make an assessment of the overall learning achieved (or not) by the students, comment on the learning of specific students she observed, always presenting empirical evidence. She may

comment on particularly relevant aspects of the lesson with regard to the proposed task, planning, materials used, introduction of the task, autonomous work, and whole-class discussion.

3. The teacher educator asks questions about the interventions of the pre-service teacher who taught the lesson and the observer, aiming to lead them to clarify or deepen issues. The teacher educator seeks to identify relevant issues (about the same topics as in 2.), to deepen the respective reasons and to lead the pre-service teachers to formulate proposals for action aimed at overcoming problems, so that they internalize that this is as a reflection process. The teacher educator may also ask questions, if relevant, about the level of attention and involvement of the students, both in the autonomous work and in the whole-class discussion.

This process of reflection was supported by looking at video recording segments of the research lesson. In addition, the pre-service teachers were asked to write a reflection, considering the lesson's flow in relation to what was planned (with aspects that "went wrong" and "went well"), describe the students' work in the different phases, indicate their learning based on empirical material (students' talk or written work) and indicate possible changes to be made to the lesson plan or the way the lesson was led, substantiating their suggestions.

The post-lesson discussion sessions followed this overall plan in a flexible way. The participants shared the evidence that they collected (Figures 4 and 5), analyzing significant episodes from the lessons and carrying out an informed discussion about specific situations.

Identification of unforeseen events

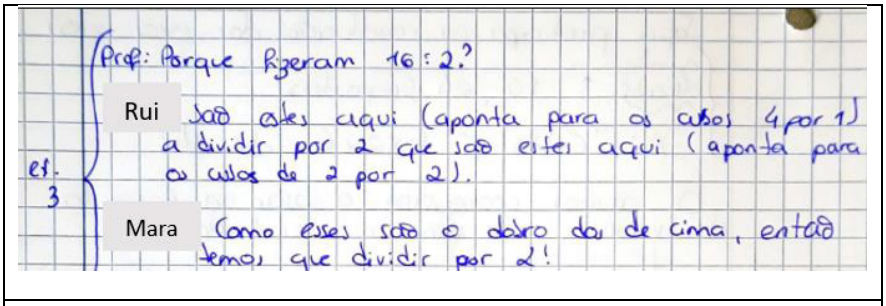
In the discussion of the first research lesson, Júlia was the first to speak:

> Júlia: I consider that the aim of the lesson was fulfilled. But it was a complicated lesson, because the students didn't want to work in the groups I formed. It was not well received. Then, in the introduction of the task, no one had any questions. Then, during the autonomous work, I realized that almost all students were having difficulties!
> Teacher educator: And what were these questions?
> Júlia: Not even I was understanding what the questions were. Because they didn't understand what "dimensions" were. I don't know why, they weren't associating the dimensions with length, width and height. Perhaps by changing the statement, it helped them. And I didn't have to intervene so much.
> Teacher educator: And change the statement how?
> Júlia: Yes... Just by putting the dimensions in the figure. They confused the width and length a lot. That's why I chose to stop everything and draw on the board. And then I identified each dimension. (S6)

Júlia began by reflecting on the difficulties that the students presented and that were not anticipated, namely concerning the organization of the groups and the statement of the task. These were unforeseen events. The pre-service teacher indicated that she had difficulty in understanding the students' questions and was only able to do so during the autonomous work phase, where she asked the students to stop their work to clarify the question on the board.

Other reflections from the lesson observations

Observing the research lessons allowed the observers to share relevant empirical material collected during the lessons. In the post-lesson discussion sessions, the participants shared this collected evidence (Figures 4 and 5), which made it possible to analyze other significant episodes from the lessons and to have a more informed discussion about specific situations.



Teacher: Why did you do 16:2?

Rui: It's these over here [points to the 4 by 1 cubes] divided by 2 which are these over here [points to the 2 by 2 cubes].

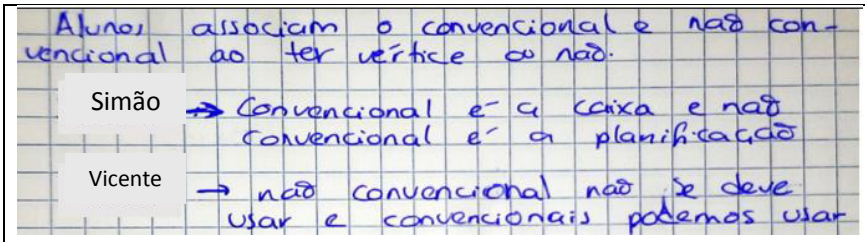
Mara: As these are double the ones above, then we have to divide by 2.

FIGURE 4 – JÚLIA'S OBSERVATION NOTES ON THE DISCUSSION OF THE TASK OF RESEARCH LESSON 2, WITH INTERVENTIONS FROM STUDENTS RUI AND MARA.

The observation notes of Júlia, regarding the whole-class discussion of task 2, shows Alda asking students to explain their answer. Students Rui and Mara have different strategies, both anticipated in the lesson planning phase and sequenced according to the pre-established criteria.

Sharing the observation notes of Júlia (Figure 5) prompted the participants to discuss a difficulty the students had that was not anticipated in the lesson planning sessions:

> Júlia: I never imagined that they wouldn't know what "conventional" was, or that their difficulty in solving the task would be that way.
> Alda: Yes, neither did I. They find it very difficult to understand certain concepts...
> Júlia: Oh yes! I saw it in my lesson, when they did not know what "dimensions" were either. (S8)



Students associate conventional and unconventional with having a vertex or not.

Simão: Conventional is the box and unconventional is the plan of the box.

Vicente: Unconventional shouldn't be used and conventional should be used.

FIGURE 5 - JÚLIA'S OBSERVATION NOTES ON THE DIFFICULTIES PRESENTED BY THE STUDENTS SIMÃO AND VICENTE DURING THE DISCUSSION OF THE TASK OF RESEARCH LESSON 2.

The sharing of the empirical material collected by the observers was complemented by the viewing of excerpts from the video recording of the research lessons, which enriched the post-lesson discussion, in which the participants focused on specific aspects of their practice.

Regarding research lesson 2, the way Alda made the introduction of the task was discussed by the group:

Júlia: I thought Alda spoke too quickly at this stage [introducing the task], but it wasn't even the worst time!
Alda: Yes, but I was very nervous here... They wouldn't hear me!
Teacher educator: Why is that?
Alda: I don't know... Maybe I shouldn't have given them the assignment straight away?!
Julia: I think that was it. Alda handed out the task statement at the same time as she was reading it and meanwhile the students were already solving the task and nobody was listening to her. You can see that here [in the video of the lesson]. And I also think that, due to her nervousness, Alda stopped several times in her speech, it wasn't fluid.
Teacher educator: Alda, but in the lesson planning, we didn't consider introducing the task like this...
Alda: No, we didn't! I never remembered that I was supposed to read the statement first, then hand it out...
Júlia: And one task item at a time!
Alda: It was really because I was so nervous, I completely forgot. And I was worried about time. (S8)

In summary, the lesson observation provided the main elements for this reflection. The two pre-service teachers draw on their observation notes and on viewing video excerpts to address elements about students' strategies and difficulties. They also refer to aspects of their teaching practice that, looking in retrospect, they regard as unsuccessful, such as the introduction of the task in the research lesson led by Alda. The quality of the observation and the observation records obtained is a very important condition for the success of the reflection of participants.

Role of previous preparation and study phase

During the post-lesson discussion sessions, the pre-service teachers drew on their prior knowledge several times, particularly on their knowledge about students and how they learn:

Júlia: I think we should have taken manipulative objects to help them with spatial visualization. We did that when we worked on perimeter and area. They measured objects in the playground, they used plasticine in another activity and I think that helps them a lot. Because first they work with the real thing and then they visualize without the object present... (S9)

The pre-service teachers also analyzed the students' difficulties, comparing those that were anticipated in the planning phase of the research lessons with those that actually arose in class:

Alda: We anticipated that they might confuse area with volume, but we didn't really talk about the possibility of them actually confusing the square with the cube. But it's very strange, because they've been working with dimensions since the first [four years] of primary school, haven't they?! (S6)

The pre-service teachers also referred to the study phase, considering the value of the documents studied during the planning phase of the research lessons:

Alda: Our students were very much in line with what we read in the articles, in that colleague's thesis... But they still managed to surprise us.
Teacher educator: In what way, Alda?
Alda: The difficulties. Because really, I don't think there were any strategies for solving the tasks that weren't in the documents studied. But there were different difficulties. (S9)

Alda also explained that this detailed study of the students' strategies and difficulties allowed her to feel more comfortable in leading class, because she could see the students' solutions and choose the ones that she wanted to bring to the whole-class discussion.

Concepts and ideas from previous experiences and knowledge were mobilized in the pre-service teachers' reflections. Also very important was the work carried out during the study phase and the planning of the research lessons, that created the basis for the lesson and enabled to review how events occurred in connection with the way they were designed and planned.

Participant involvement

The participants were very involved and committed in all the lesson study sessions, particularly in the post-lesson discussion and final reflection sessions, when they tried to identify aspects of the research lessons that did not go well and to suggest ways of improving them:

> Alda: For example, that situation where I introduced the task in that way... I agree that I didn't do it well, but I didn't even realize it at the time, only afterwards when I spoke to Júlia and she drew my attention to it! (S9)

In the reflection session, the participants took stock of all the work carried out in the lesson study. In this session, Alda participated more actively in the discussion comparing to the two former lesson studies in which she participated, probably because it was the first time she led a research lesson and she was more aware of the whole process.

The participants' involvement was very important for their reflections. To achieve depth and relevance in the reflection it is necessary that they assume an active stance of stiving to identify issues, their causes and possible ways of overcoming them.

As with what happens in all other lessons, much of what occurs depends on the participation of the students. In some cases, it seems that they strive to make things difficult for the teachers. That was not the case in the two lessons we report here, in which the students participated actively working in the tasks proposed and participating in the whole-class discussion.

The role of the teacher educator

Based on the post-lesson discussion structure presented earlier, the teacher educator organized the post-lesson discussion sessions. She led the sessions by organizing the moments of discussion and analysis by phase of the exploratory lesson, in order to ensure that all moments of the lesson were covered in the discussion and also to support the participants' interventions along a guiding and sequential line of their ideas. During the sessions, the teacher educator asked open-ended and targeted questions in order to promote critical analysis from the pre-service teachers, encouraging their reflection on aspects such as the defined learning aim, as well as the design of the tasks proposed to the students and the students' learning:

> Teacher educator: Alda, as an observer, do you feel that the aim of the lesson was fulfilled?
> Alda: Yes, and I could be sure of that when Júlia led the final synthesis.
> Teacher educator: And why at that particular moment?
> Alda: They participated a lot... They managed to define the concept of volume, which was the lesson aim.
> Teacher educator: What do you think would be important to change in the lesson?
> Júlia: To form different groups, because of their behavior.
> Teacher educator: What about specific aspects, like the tasks? Would you adapt or change the tasks?
> Júlia: Perhaps, in the one where they showed that they didn't know what the dimensions were... Perhaps I would put in the task statement that the length was *x*, the width was *y* and the height was *z*. But then I would also condition them. We would already give them part of the answer. (S6)

In the final part of this dialogue, the teacher educator invited the pre-service teachers to critically reflect on the design of the tasks, through direct questions, which led Júlia to think about changes she would make to the wording of task 1, in order to support the students in the difficulty they showed in understanding the notion of dimension.

The role of the teacher educator in this lesson study was determinant from beginning to end and also in the post-lesson discussion. She led the discussion following the planned structure. The existence of a

structure to lead the discussion was an important resource for the teacher educator, proving her direction about what to do at every step of the discussion. Besides giving the word to the pre-service teachers, the teacher educator asked targeted and open questions about the different elements that frame the lesson. In addition, she assumed a friendly and constructive stance in order to lead the pre-service teachers to feel comfortable and willing to consider sensitive issues.

1.8 Share LS report **[Link to General definition]**

Two sessions were scheduled to prepare the dissemination of the results of this lesson study. This will be reported by pre-service teachers Júlia and Alda at a national seminar about lesson studies in initial teacher education. The pre-service teachers will describe each phase of the lesson study from their perspective and share their experience and how they perceived their participation in this formative process, highlighting their main ideas that they learned and the challenges they faced.

1.9 Feedback on the lesson study in ITE: strengths and weaknesses of this variant

This case study is an example of School Hybrid LS (section 3.1.3). The choice for this format was due to the conditions of the institutional context. The teacher educator was the supervisor of the two pre-service teachers in their practicum, and she had the opportunity of carrying out a LS. The pre-service teachers were at school several days a week but went also to the university to have meetings with the supervisor. In the school they had to observe lessons taught by the mentor teacher but also to teach lessons in her class. This created the conditions for carrying LS sessions at the university and teaching research lessons at the school.

A major strength of this work is that it is carried out in the framework of the institutional context, not requiring great adjustments to the work of the teacher educator and of the pre-service teachers. In addition, the pre-service teachers have the opportunity to prepare, teach ad reflect on two research lessons in a deeper way that they would do without the existence of the LS. This was another strength of this activity. The fact that the LS group was rather small and that there was time for a detailed work together, enabled very close relationships between the three participants, and that was also a strength.

The major weakness was that the mentor teacher from the school was not available to participate in the LS work, for personal constrains. The active participation could have enriched the discussions, given her experience and knowledge of the pupils in the class.